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Total No. of Questions : 09

B.Tech. (Sem.–2)
ENGINEERING MATHEMATICS-II
Subject Code : BTAM-102 (2011 Batch)
Paper ID : [A1111]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY.**
2. **Attempt any FIVE questions SECTION - B & C.**
3. **Select at least TWO questions from SECTION - B & C.**

SECTION-A (10 × 2 = 20 Marks)

1. (a) Find the general value of log $(-1 + \sqrt{3}\ i)$.

(b) Solve the differential equation $\frac{d^3y}{dx^3} + y = 0$

(c) Under what conditions on 'a, b, c and d' ,the differential equation $(a\sinh x\cos y + b\cosh x\sin y)dx + (c\sinh x\cos y + d\cosh x\sin y)\,dy = 0$, is exact ?

(d) Find all the roots of $(-8i)^{1/3}$.

(e) Test for what values of 'k' the set of vectors {(k,1,1), (0,1,1),(k,0,k)} is linearly independent.

(f) Examine the convergence / divergence of the series $\sum_{n=1}^{\infty}\frac{x^n}{(2n)!}$

(g) Test the absolute convergence of the series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\log n)^2}$

(h) Test whether the matrix $\begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$ is diagonalisable or not ?

(i) For what values of 'k' , the system of equation

$x + y + z = 2$; $x + 2y + z = -2$; $x + y + (k – 5) z = k$,

has no solution.

(j) Express $\cos 6\theta$ in terms of powers of $\cos\theta$.

SECTION-B **(4 × 5 = 20 Marks)**

2. (a) Find the general solution of the differential

$(3x^2y^3\ e^y + y^3 + y^2)\ dx + (x^2y^3\ e^y\ -xy)dy = 0$

(b) Obtain the general solution of the equation $y'' + 3\ y' + 2\ y = \sin (e^x)$, by using method of variation of parameters .

3. (a) Solve the following simultaneous differential equation

$$\frac{dx}{dt} + y = \sin t,\ \frac{dy}{dt} + x = \cos t,\ y(0) = 0,\ x(0) = 2$$

(b) Find the complete solution of the differential equation

$(x + 1)^2\ y'' + (x + 1)y' + y = \sin(2\ \log(1 + x))$.

by using operator method.

4. (a) Solve the differential equation $x(\frac{dy}{dx} + y) = 1- y$

(b) Find the particular solution of the differential equation $y'' + a^2y = \sec ax$

5. An L-C-R circuit with battery e.m.f '$E \sin pt$ 'is tuned to resonance so that $p^2 = \frac{1}{LC}$. If initially the current i and the charge q be zero , then show that for small value of $\frac{R}{L}$, the current in the circuit at time t is given by $\frac{E}{2L} t \sin pt$.

SECTION-C **(10 × 2 = 20 Marks)**

6. (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$

7. (a) Test the consistency of the system of equations
$x + 2y - z = 3$; $3x - y + 2z = 1$; $2x - 2y + 3z = 2$; $x - y + z = -1$, and if consistent, then solve it completely.

(b) Reduce the matrix

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

to normal form and hence find its rank.

8. (a) Discuss for what values of' x' does the series

$$\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \ldots\ldots \infty, \text{ converge/diverge.}$$

(b) Examine the convergence/diverge of the series

$$\sum_{n=1}^{\infty}[(n^3+1)^{\frac{1}{3}} - n]$$

9. (a) Use Demoivre's theorem to find all the roots of the equation

$$z^4 - (1-z)^4 = 0$$

(b) Find the sum to infinity of the series

$$1 - \frac{1}{2}\cos\theta + \frac{1.3}{2.4}\cos 2\theta - \frac{1.3.5}{2.4.6}\cos 2\theta + \ldots\ldots\ldots, (-\pi < \theta < \pi).$$